PENNSTATE

Applied Research Laboratory

Capturing Uncertainty

Uncertainty in Propagation Loss Calculations

(Note: Narrative of presentation is available after the "modeling improvements" slide.

Diana McCammon

Applied Research Laboratory

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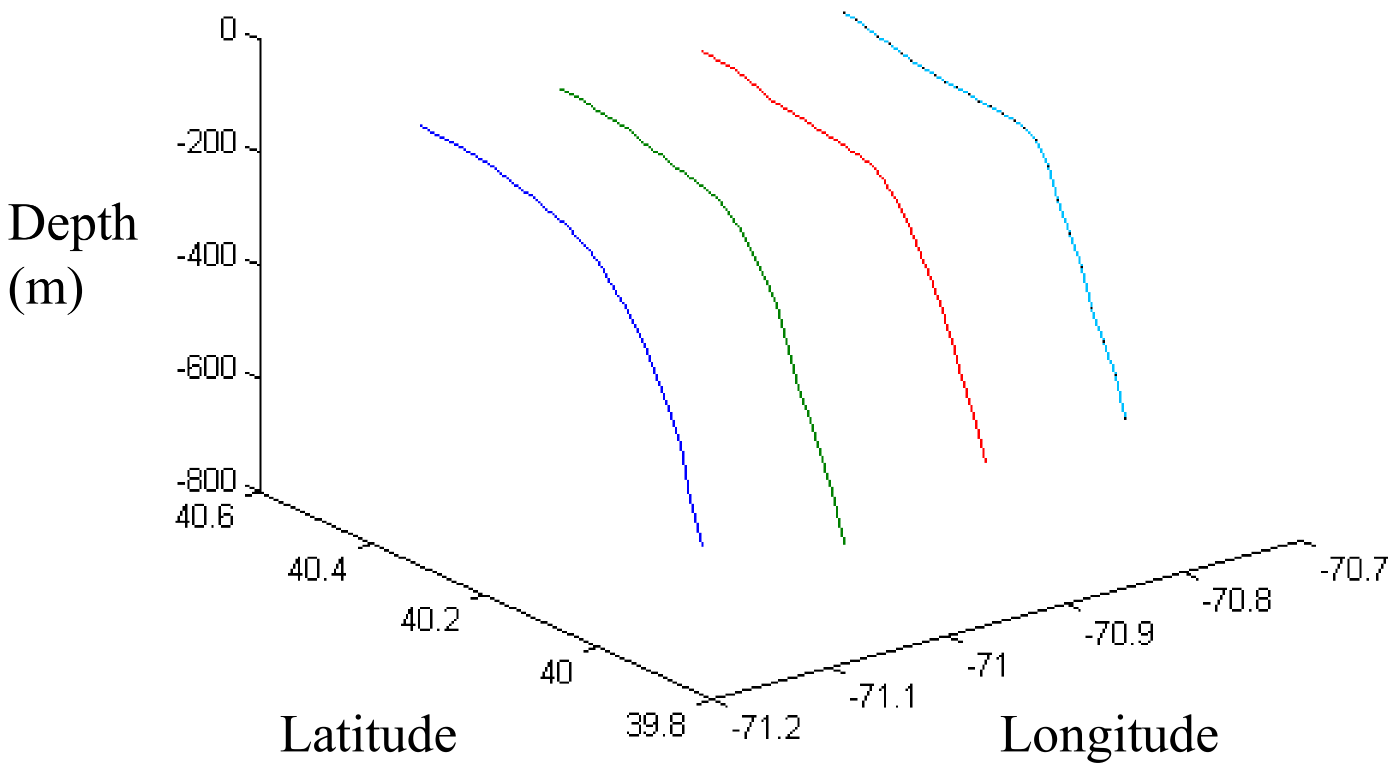
Illustrative Problem

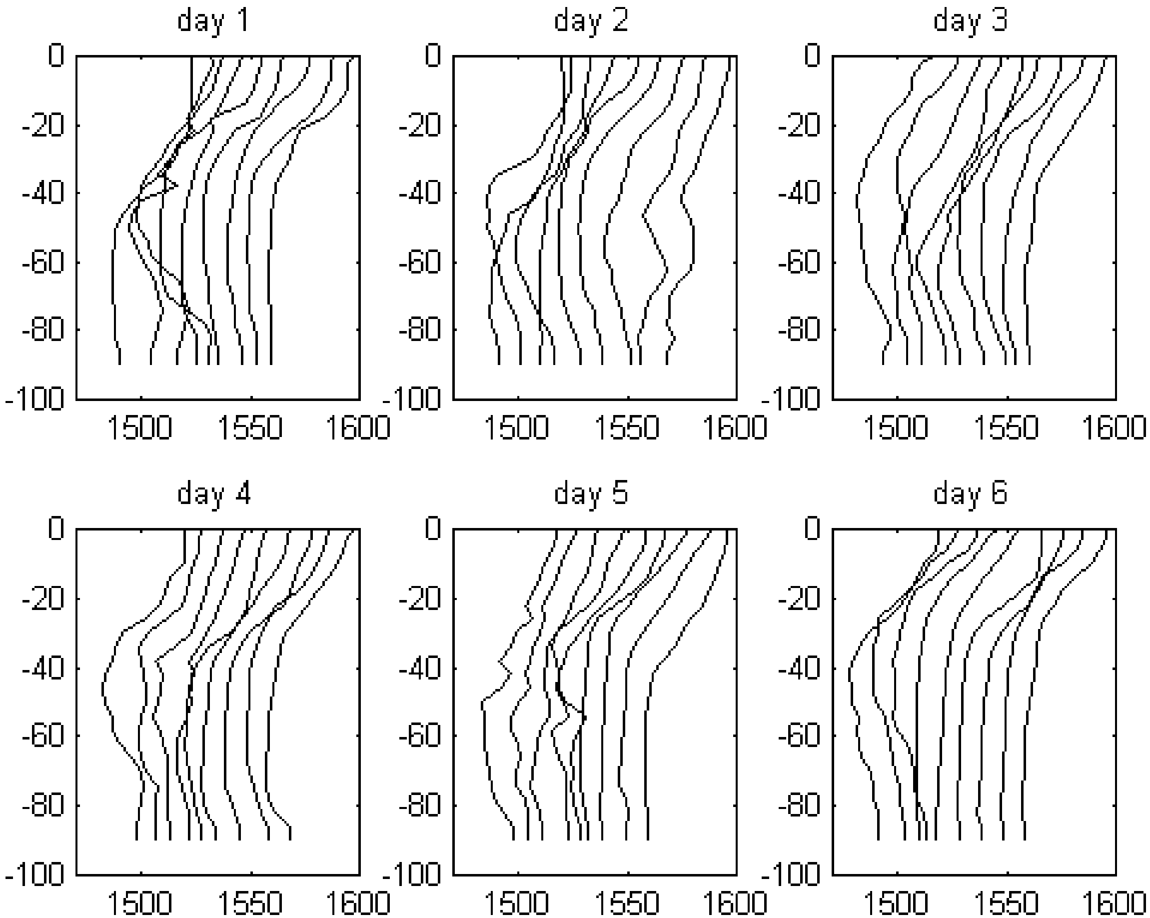
Sources of /Transport of Uncertainty

- External to the propagation model
  - Selecting range and depth grid for input data
  - Converting temperature to speed
  - Determining required output resolution

Primer bathymetry on 4 lines of measurement



SVP changes over the 6 day period on westline



Profiles shifted by 10m/sec for display

Range separation is 2.4km

Illustrative Problem
Sources of /Transport of Uncertainty

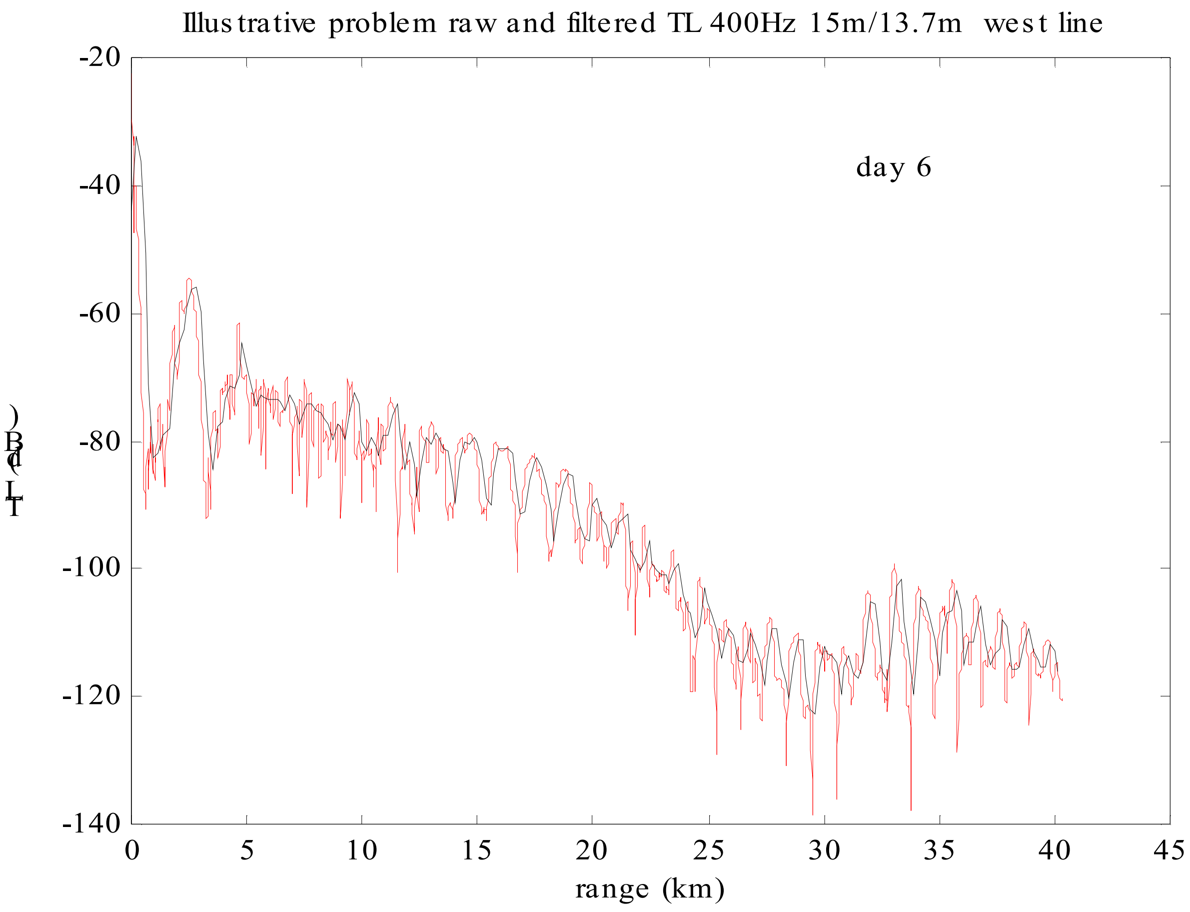
- Internal to the PE model
  - Smooth profiles over depth
  - Smooth impedance interface into bottom
    - 0-10m   0.08 dB/🖰 1650-1675 m/sec 1.7gm/cc
    - 10-150m 1.0 dB/ 🖰 1675-1750 m/sec 1.9-2.0gm/cc
  - Determine range step for adequate convergence
    - 18.5 m (5 🖰)
  - Interpolation to receiver depths if not on grid

Illustrative Problem
Sources of /Transport of Uncertainty

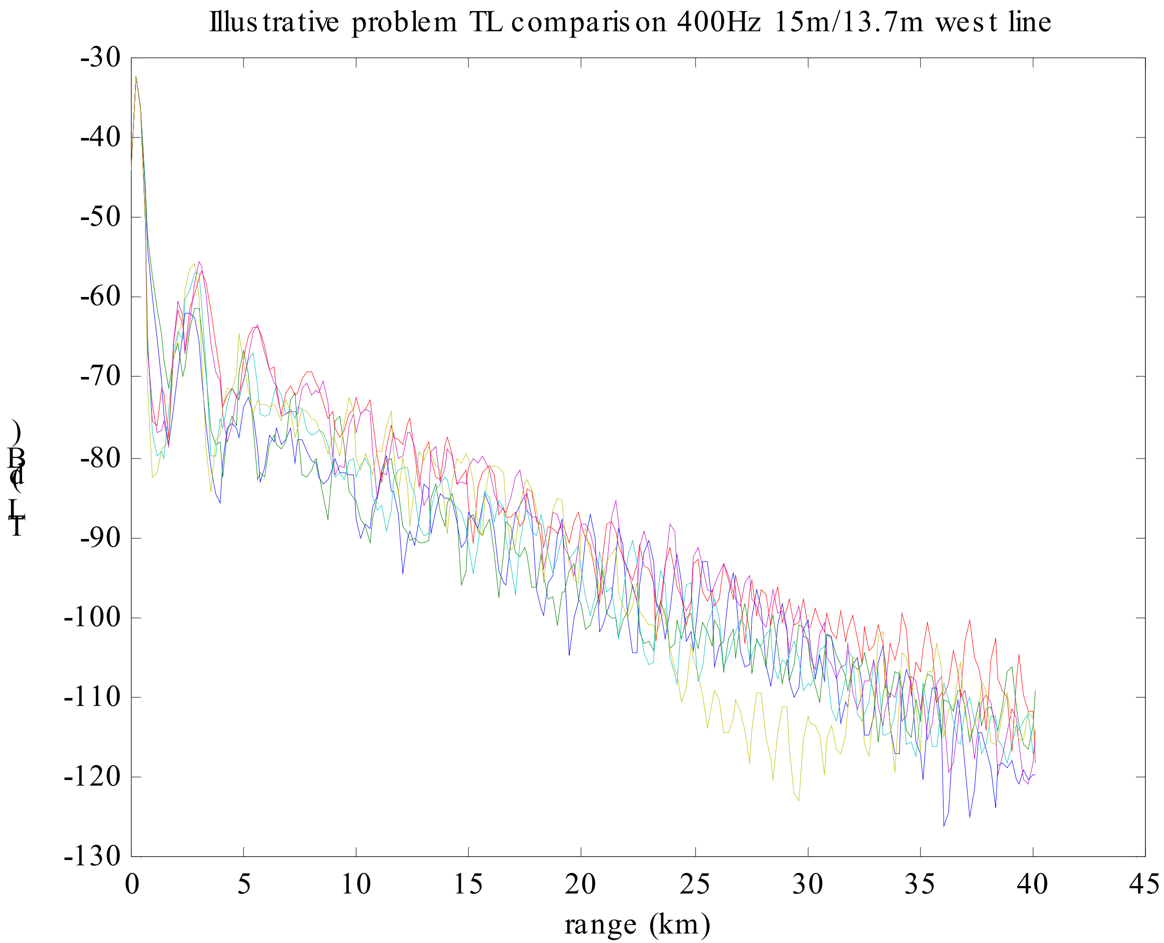
- Post computation
  - Range averaging of intensity to dampen coherent oscillations (.23nmi triangular filter)

- Results expressed as mean and standard deviation of TL in dB over 6 days

Range averaging- Raw and filtered TL for day 6 on the westline

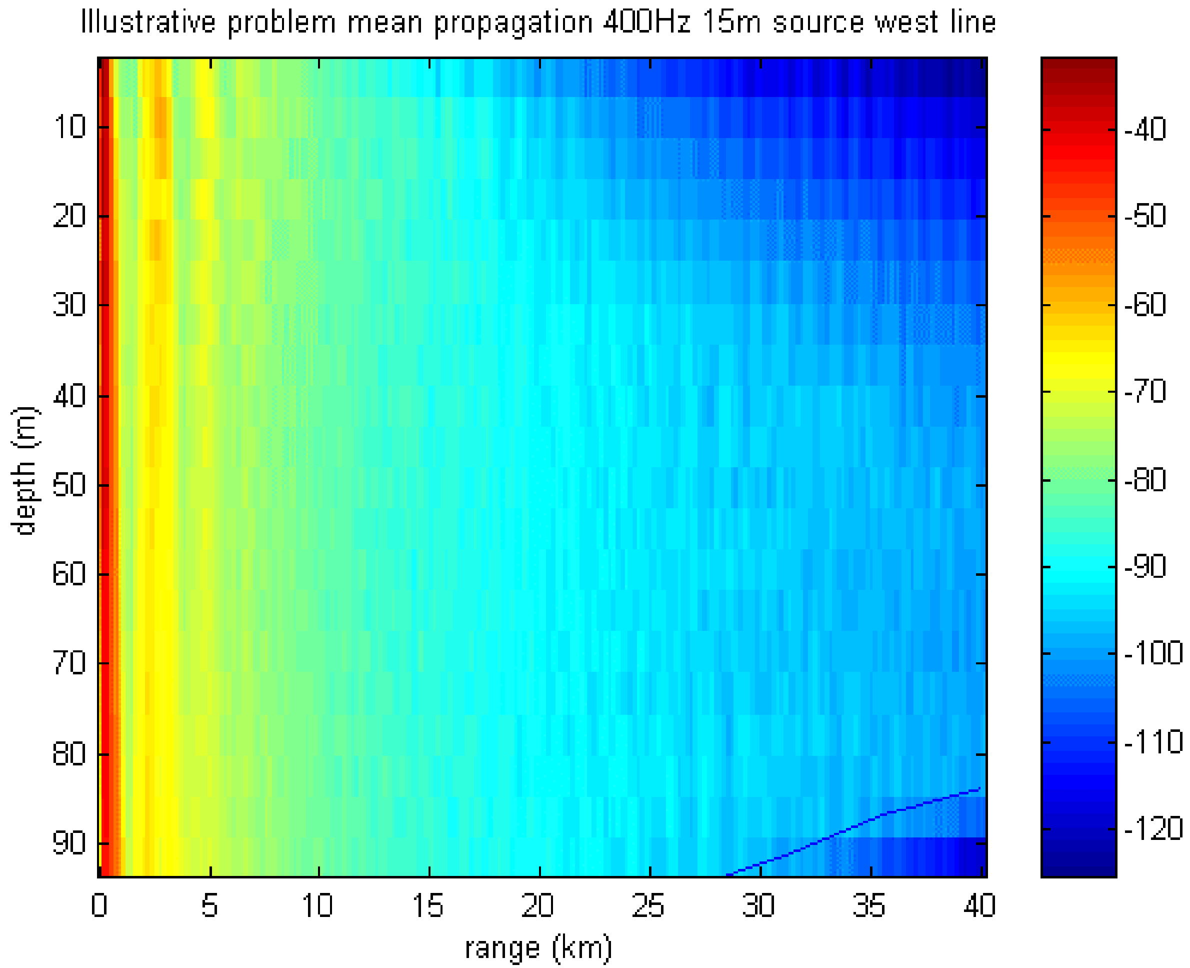


Variability - Smoothed TL for each of 6 days on westline at receiver depth 13.7m



Mean TL over all 6 days on westline

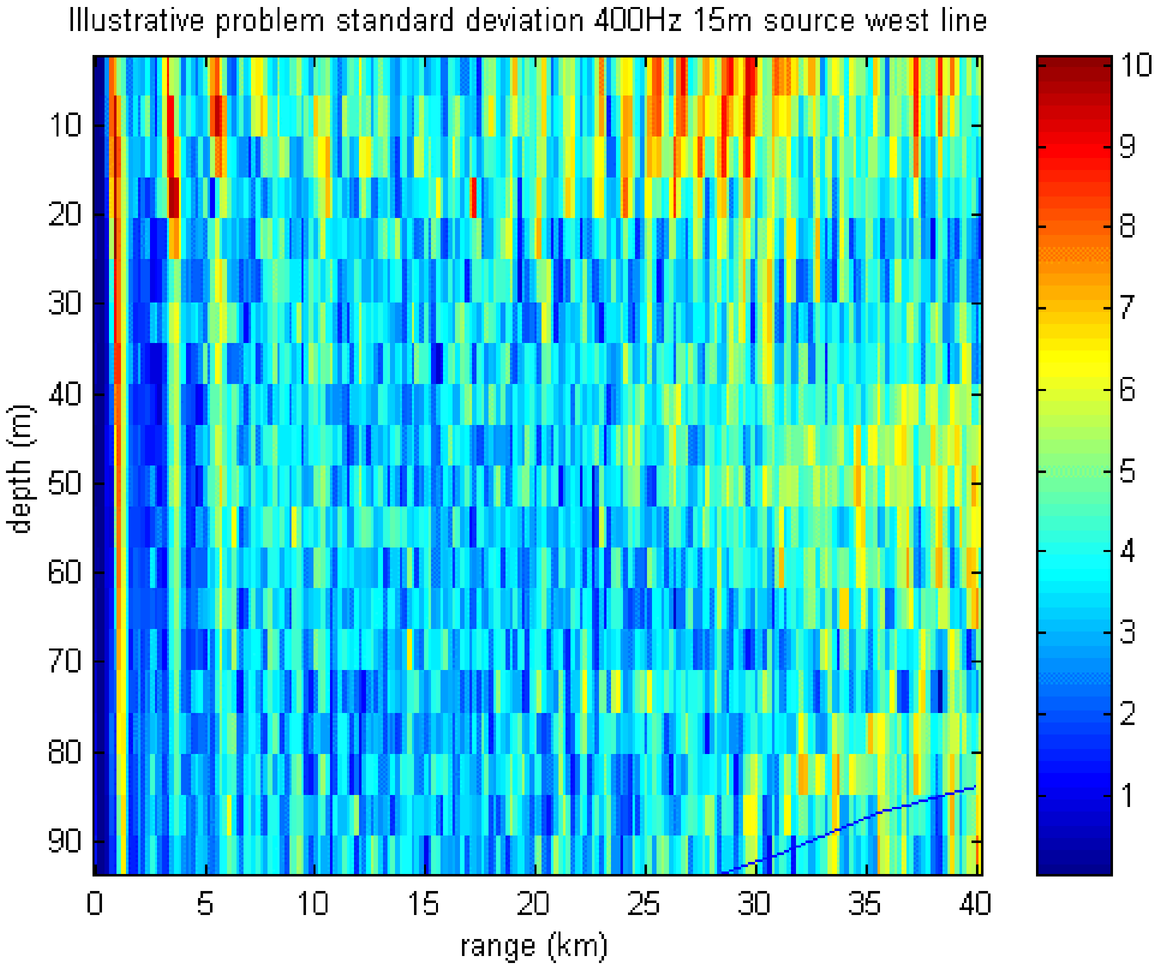
full field display limited to first 90m



Portion of bathymetry at shallow end shown by line

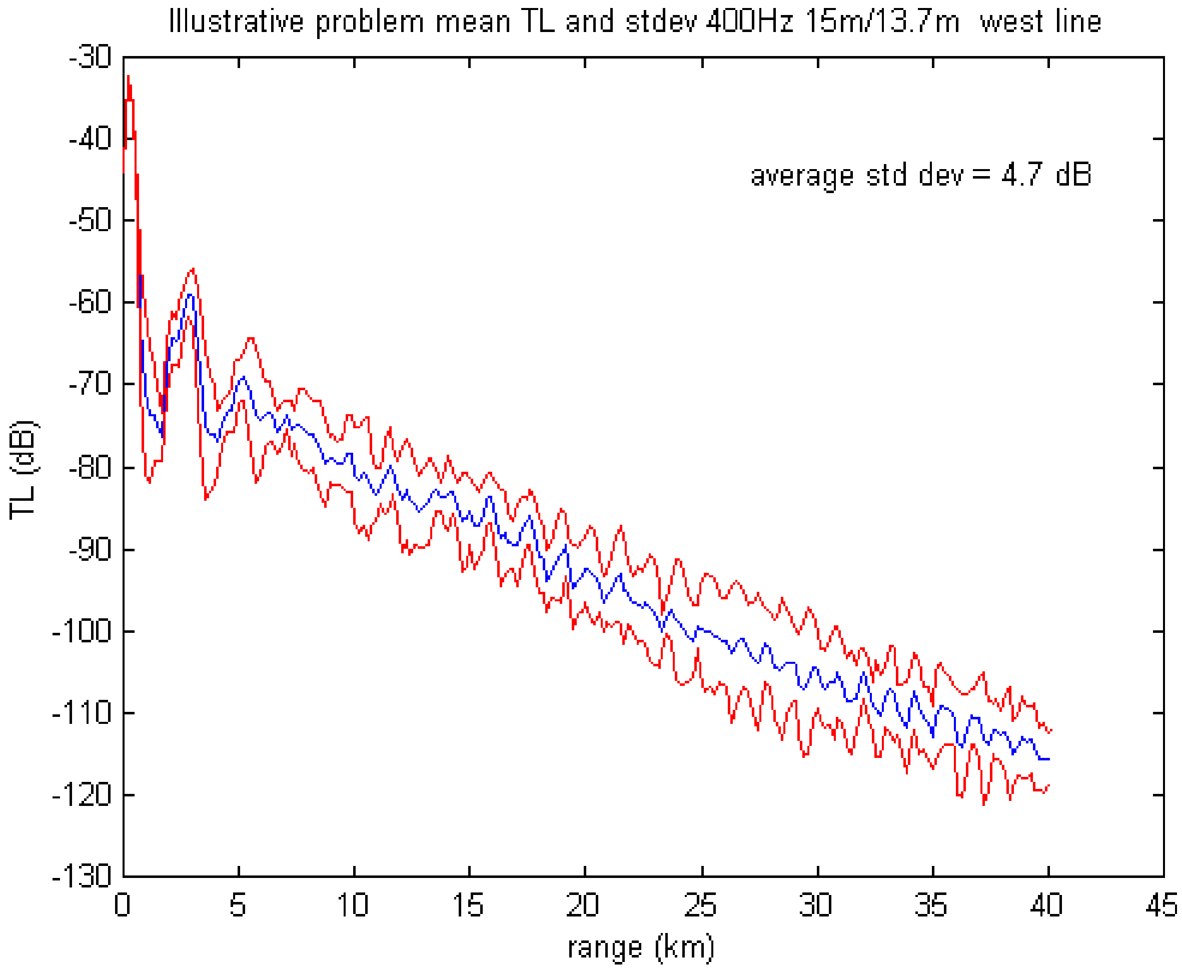
Standard Deviation of TL over all 6 days on westline

full field display limited to first 90 m



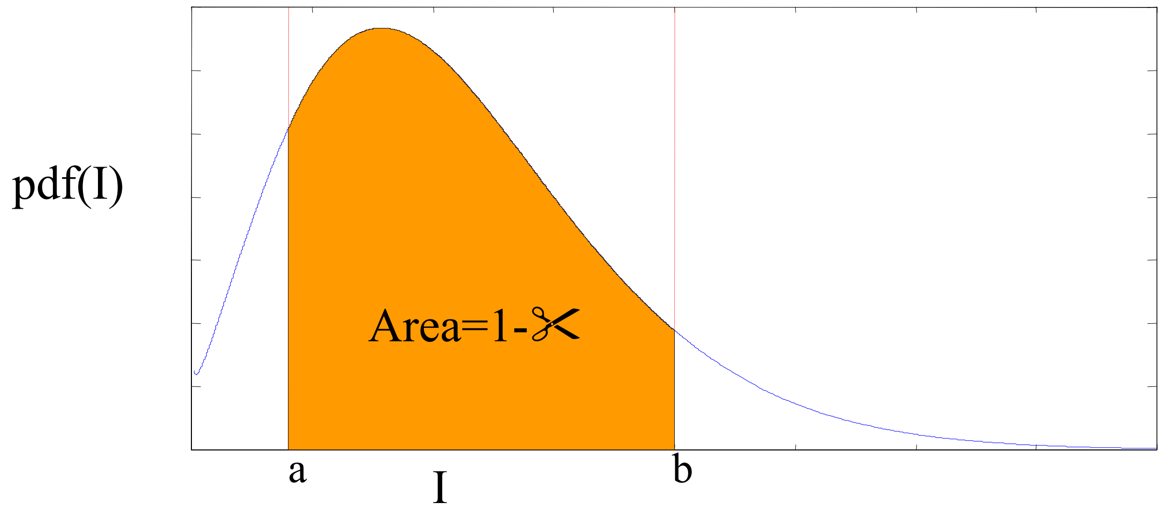
Portion of bathymetry at shallow end shown by line

Variability - Smoothed Mean TL on westline at receiver depth 13.7m with +- 1 standard deviation



Confidence bounds and Uncertainty in the PE

Assume the intensity I varies randomly between two bounds with a confidence of 1-α



The probability that I is outside either bound is α/2

$$P\{I < a\} = P\{I > b\} = \alpha / 2$$

Typical confidence levels for Navy applications

(95%=2 standard deviations, 68%=1 standard deviation)

Concept development	50%	$\alpha = 0.5$
Design / tradeoff	75%	$\alpha = 0.25$
Deployment	75-90%	$\alpha = 0.25 - 0.1$
Operation	90%	$\alpha = 0.1$

Propagation of Random Uncertainty

Let the intensity be a squared Taylor series expansion of the pressure about the random sound speed x.

$$I(r,z,x) = (p_0 + x\,(\partial p / \partial x))^2$$

x = fractional random variation of input speed, e.g. 1.5 / 1500 m/sec. The derivative of the pressure with respect to the speed randomness is found by replacing C by C(1+x) .

Then the probability that the intensity remains within bounds [a,b] with confidence 1-alpha is

$$P\,\{I < a\} = P\,\{I > b\} = \alpha/2$$

Since x is a Gaussian random variable with variance $\sigma_x^2 / x_0{}^2$ the confidence bound equation can be solved with Normal Probability Integrals Φ

$$\Phi(x_0 \frac{-p_0 + \sqrt{a}}{\sigma_x \partial p / \partial x}) - \Phi(x_0 \frac{-p_0 - \sqrt{a}}{\sigma_x \partial p / \partial x}) = \alpha / 2$$

$$\Phi(x_0 \frac{-p_0 - \sqrt{b}}{\sigma_x \partial p / \partial x}) + 1 - \Phi(x_0 \frac{-p_0 + \sqrt{b}}{\sigma_x \partial p / \partial x}) = \alpha / 2$$

For bounds, solve for a and b given α and σ_x

For input sensitivity, solve for σ_x given a,b and α

To obtain the $\partial p / \partial x$, take the derivative of the PE operator

Let $b = k_0(n-1) + i\alpha$

$$p(r+dr,z) = e^{ibdr} F^{-1}\{e^{is^2 dr/2k_0} F\{p(r,z)\}\}$$

$$\frac{\partial p(r+dr,z)}{\partial x} = (i\frac{\partial b}{\partial x}dr)p(r+dr,z) + e^{ibdr} F^{-1}\{e^{is^2 dr/2k_0} F\{\frac{\partial p(r,z)}{\partial x}\}\}$$

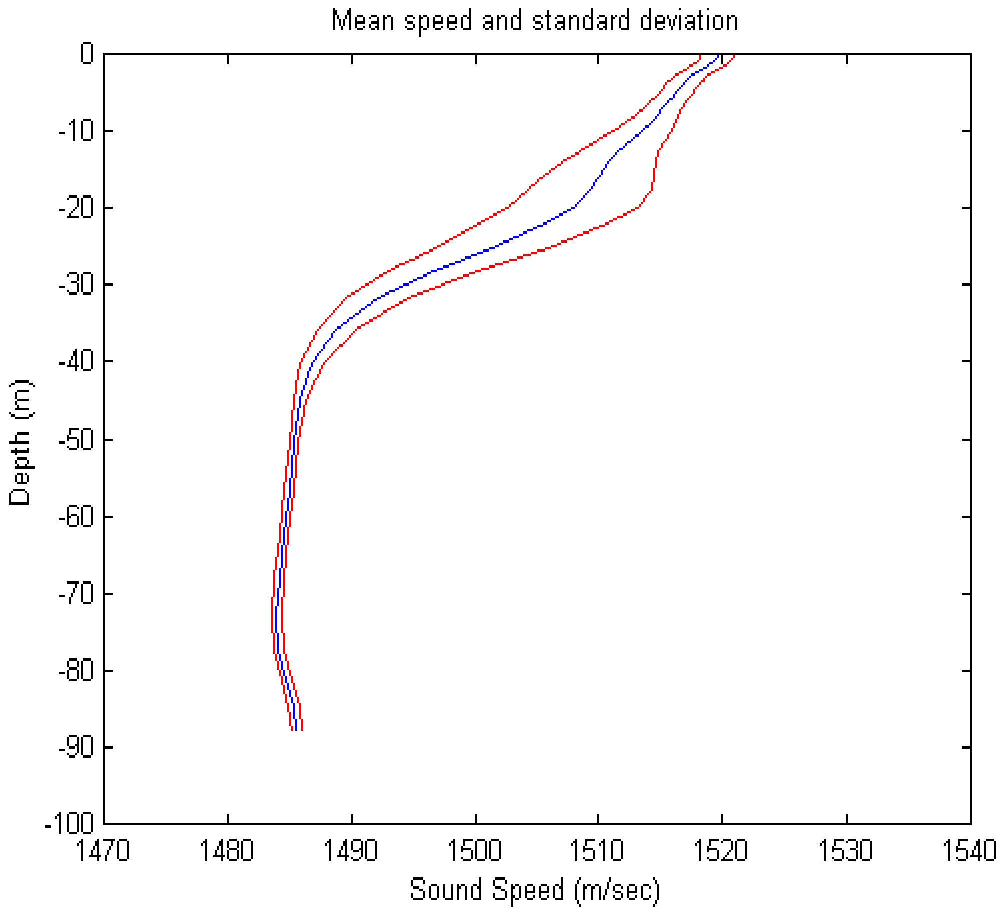
for sound speed variations,

$$\partial b / \partial x \;= -k_0 (C_0 / C)^2 / n$$

Finally, choosing an confidence level of 68% to produce the standard deviation of the intensity, we compute the effect of each random sound speed profile point individually and then sum the squares

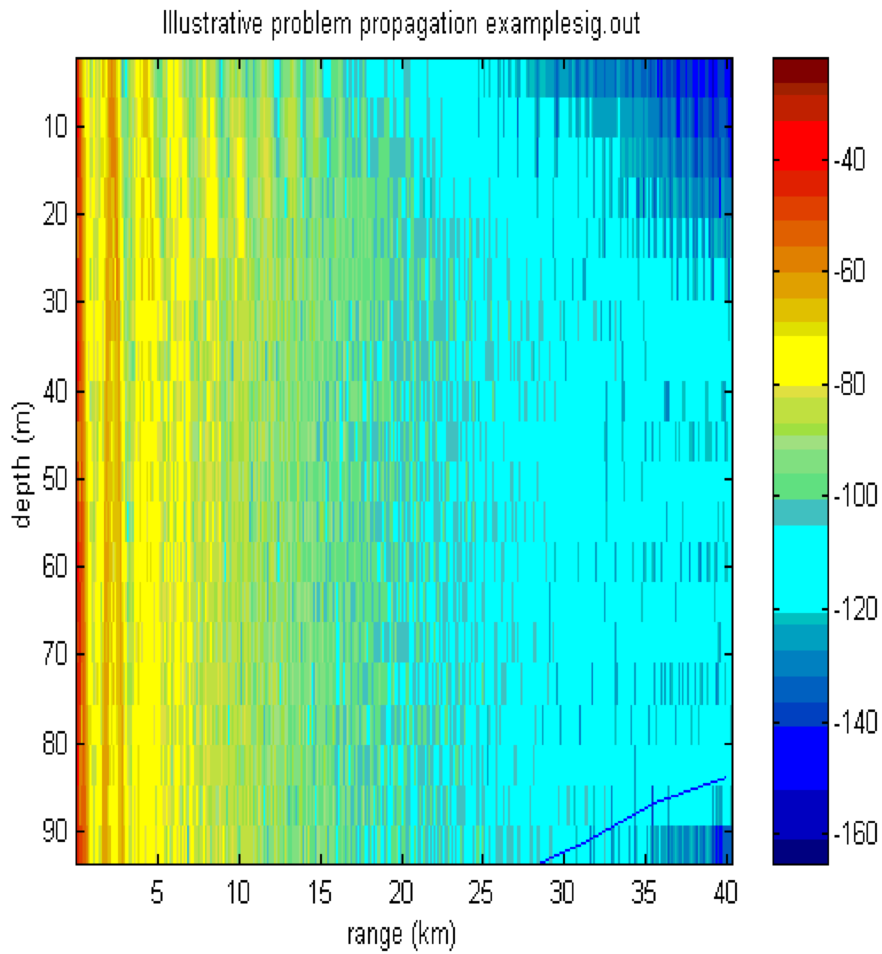
The results are $$\sigma_I = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots}$$

Assumption of independent random speed variations at each depth - mean westline

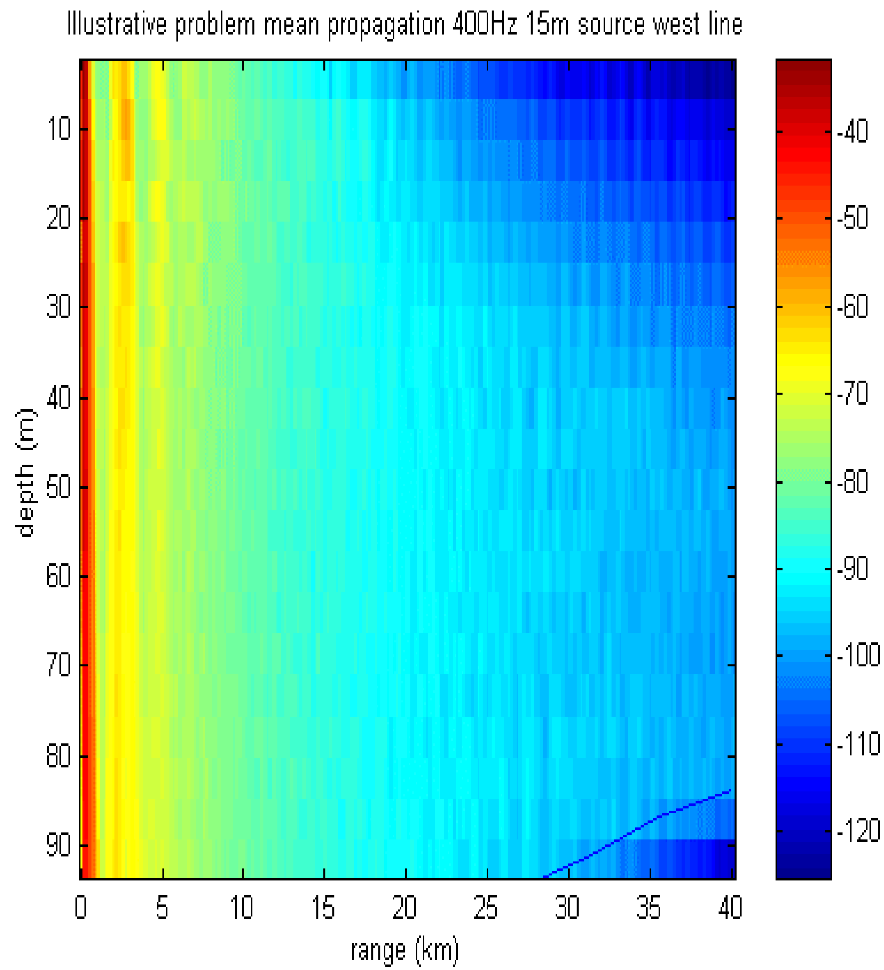


Comparison of uncertainty and variability - Mean TL

6 days of data, west line, 15m receiver

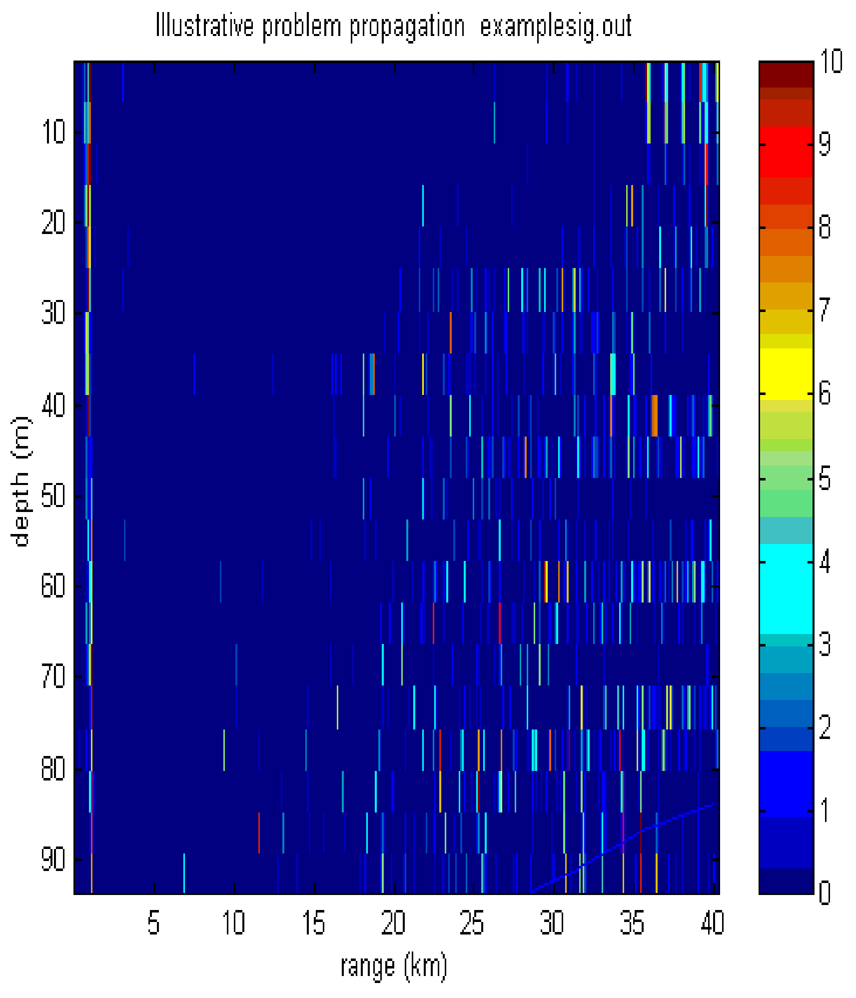


Random Uncertainty

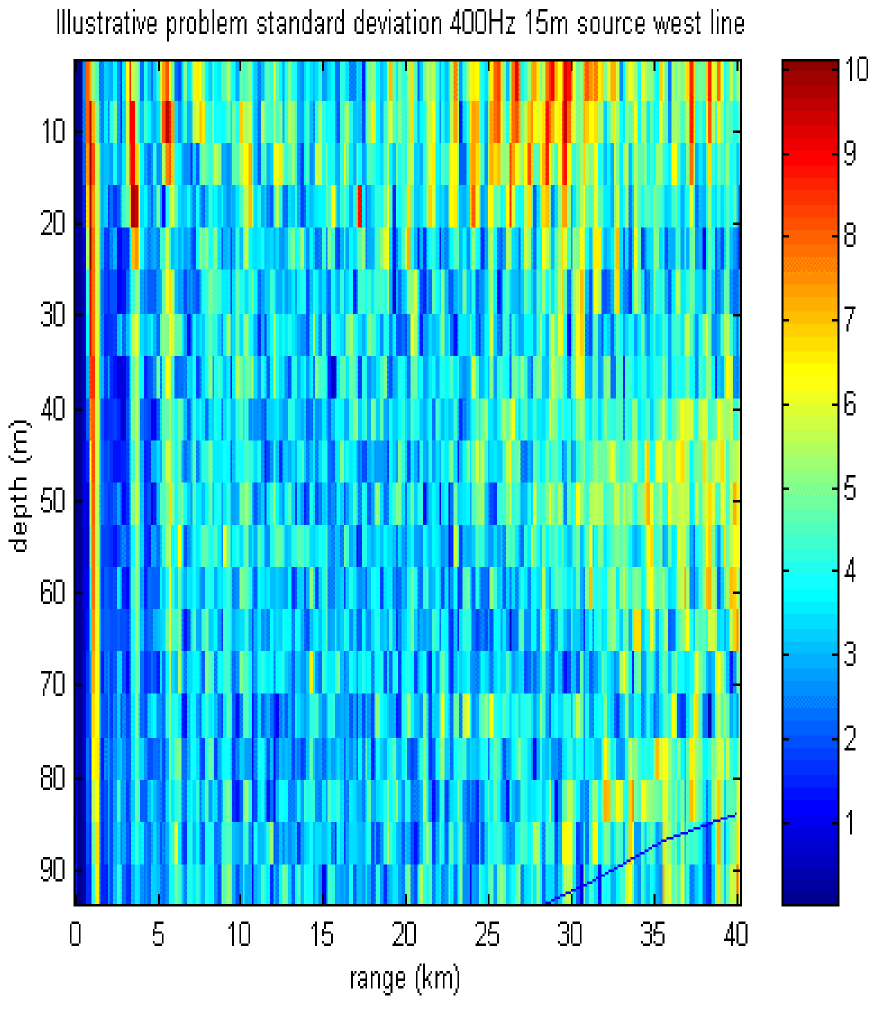


Measured Variability

Comparison of uncertainty and variability
Standard Deviation of TL 15m receiver



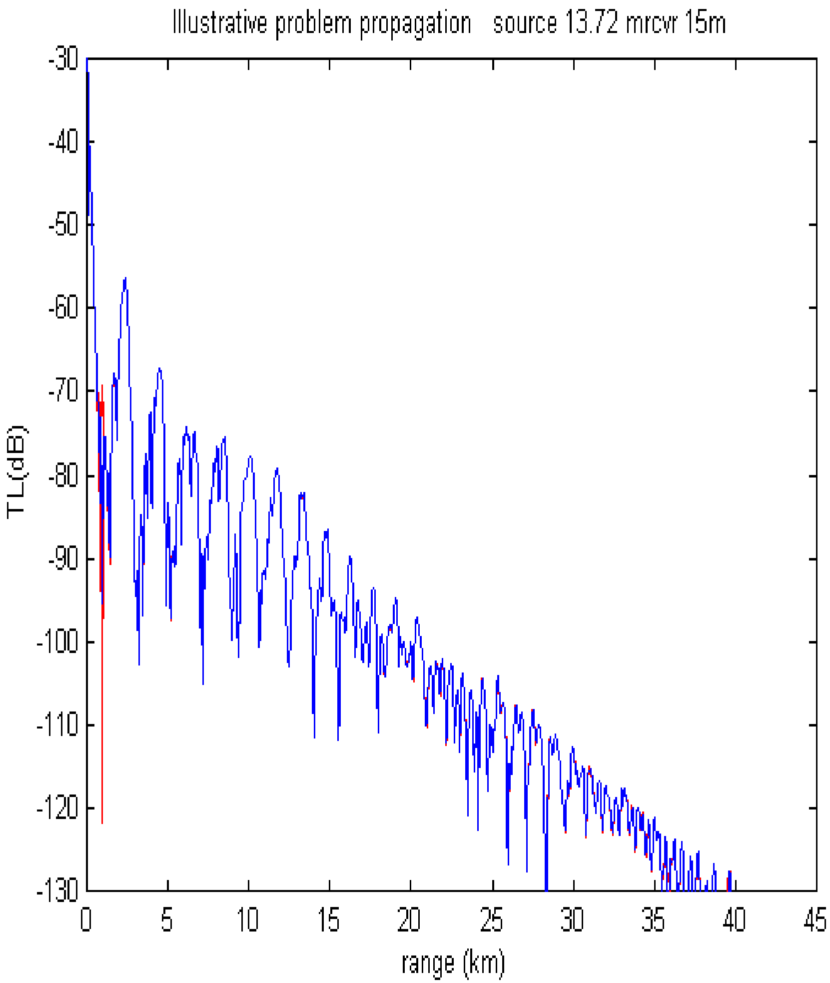
Random Uncertainty



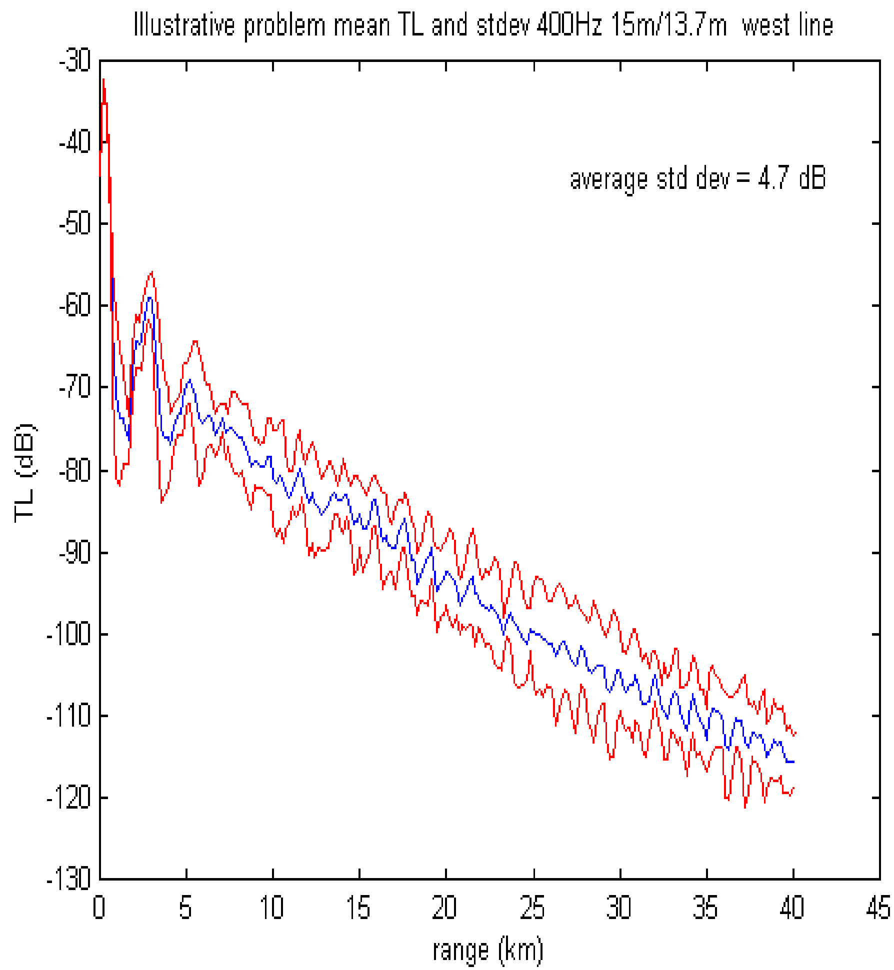
Measured Variability

TL and Sigma vs range

13.7m source/ 15m receiver

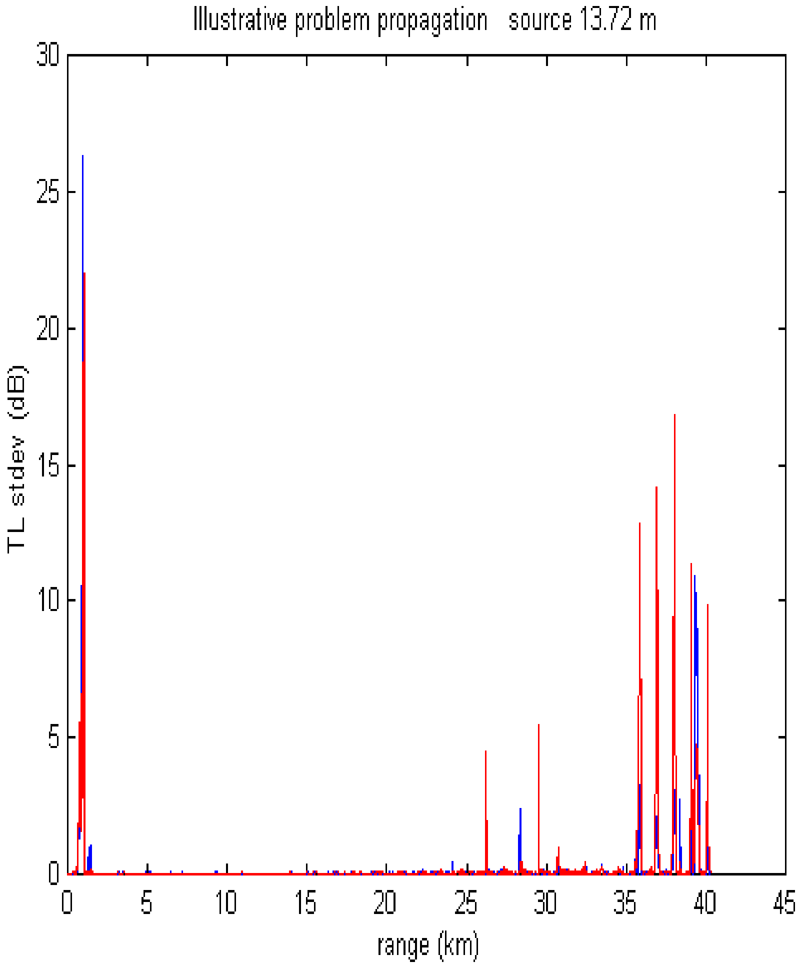


Random Uncertainty

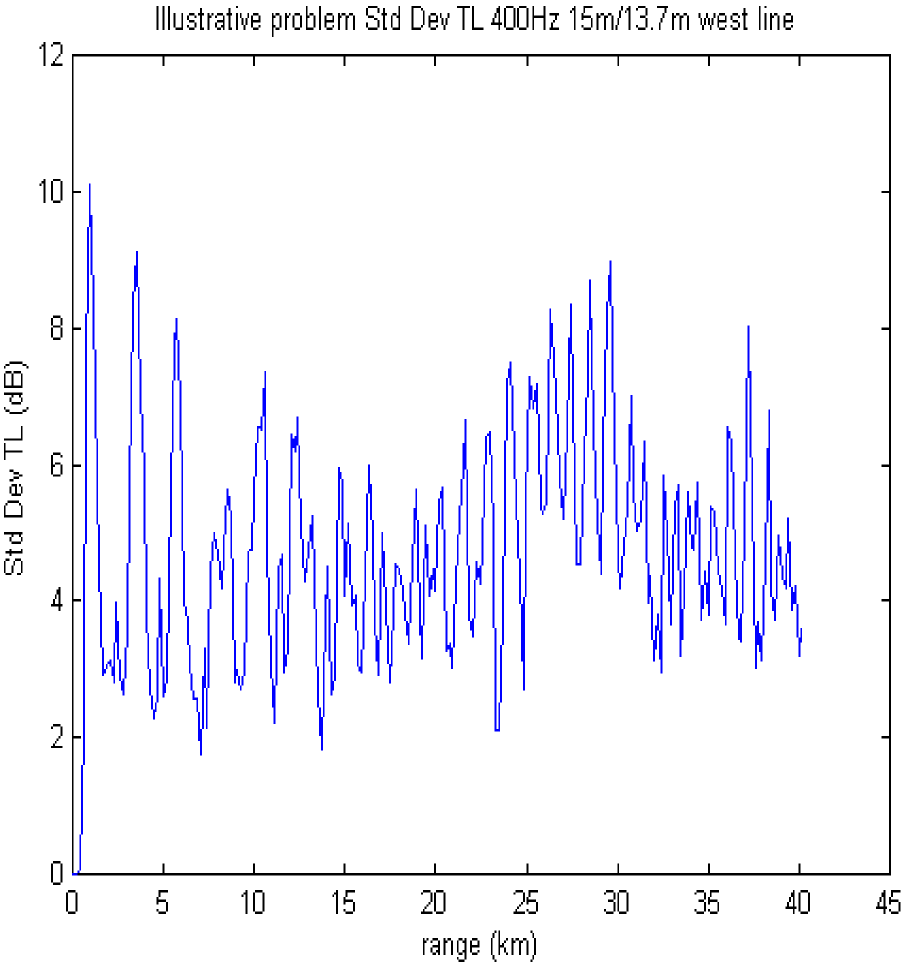


Measured Variability

Sigma vs range 13.7m source/ 15m receiver

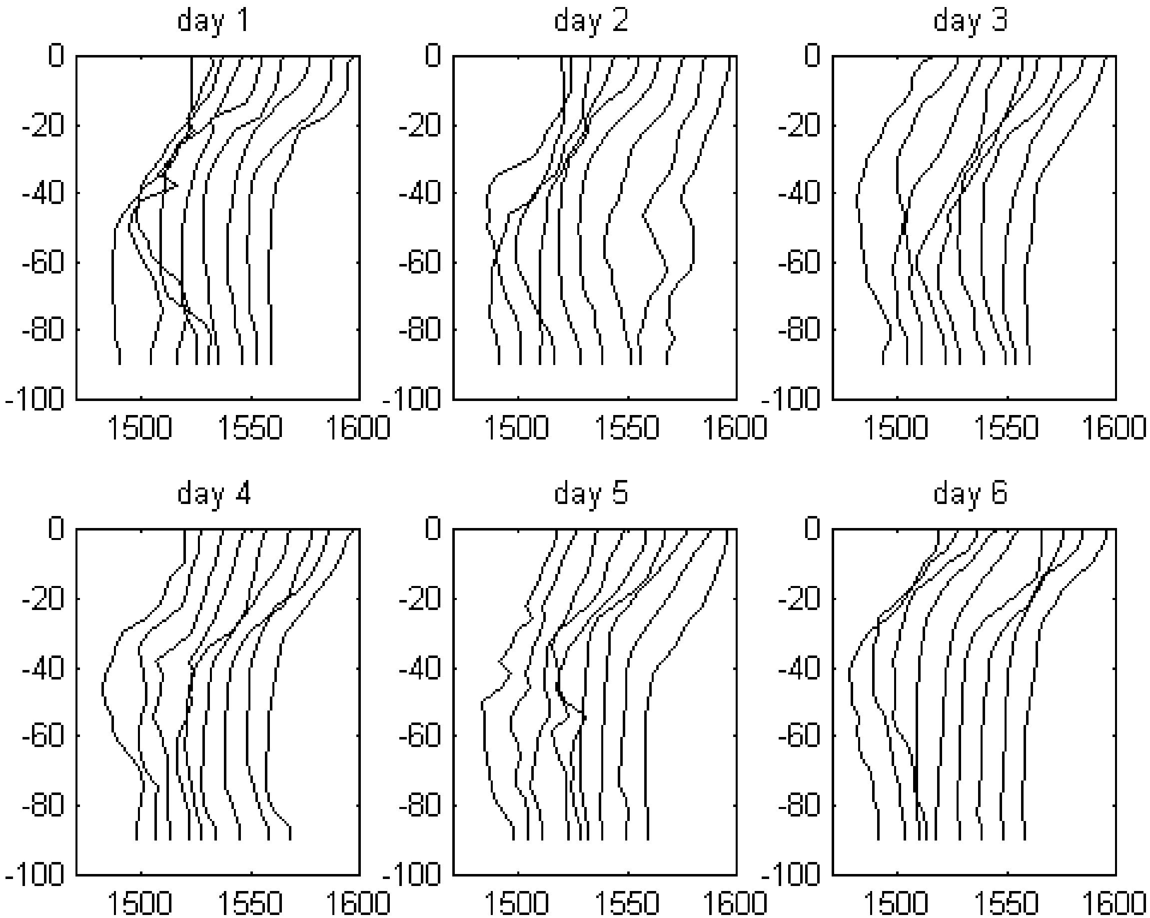


Random Uncertainty



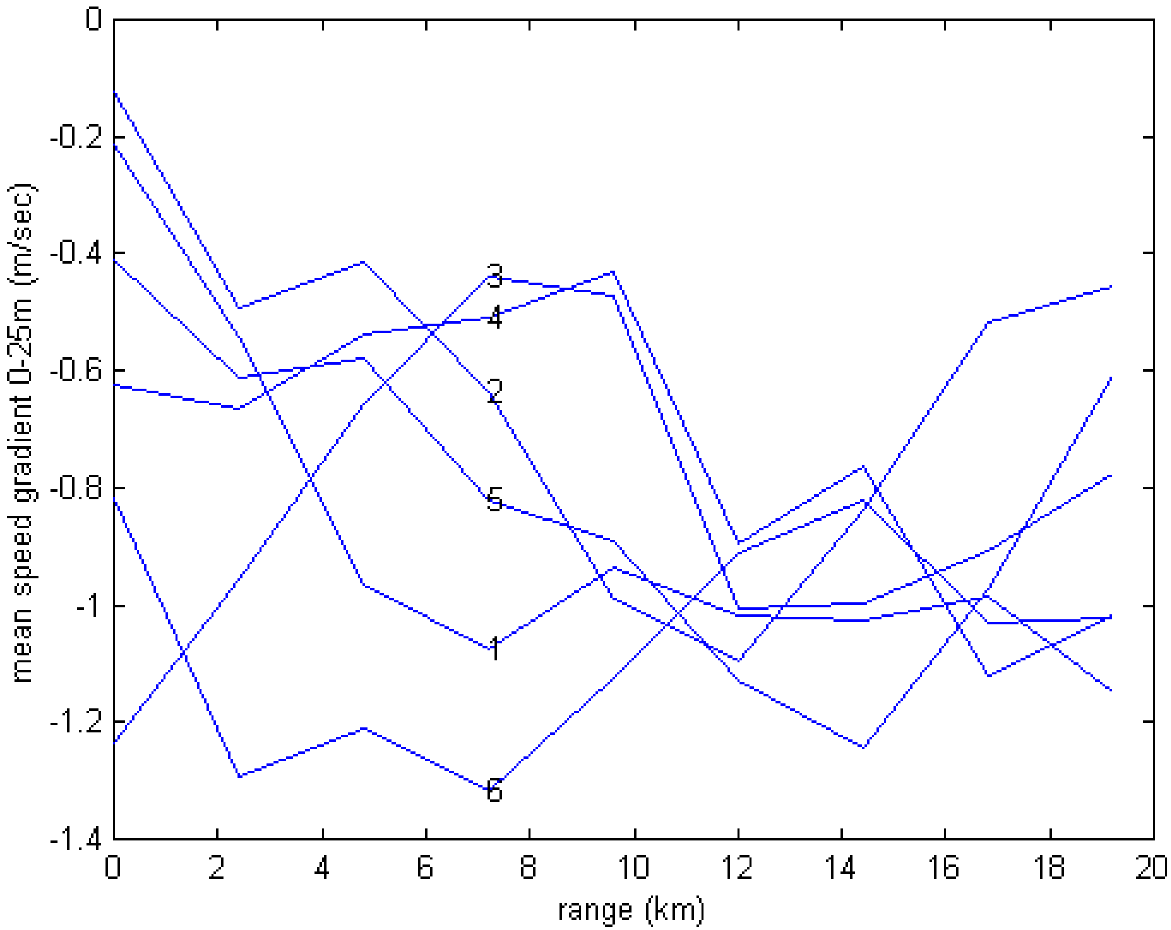
Measured Variability

SVP changes over the 6 day period

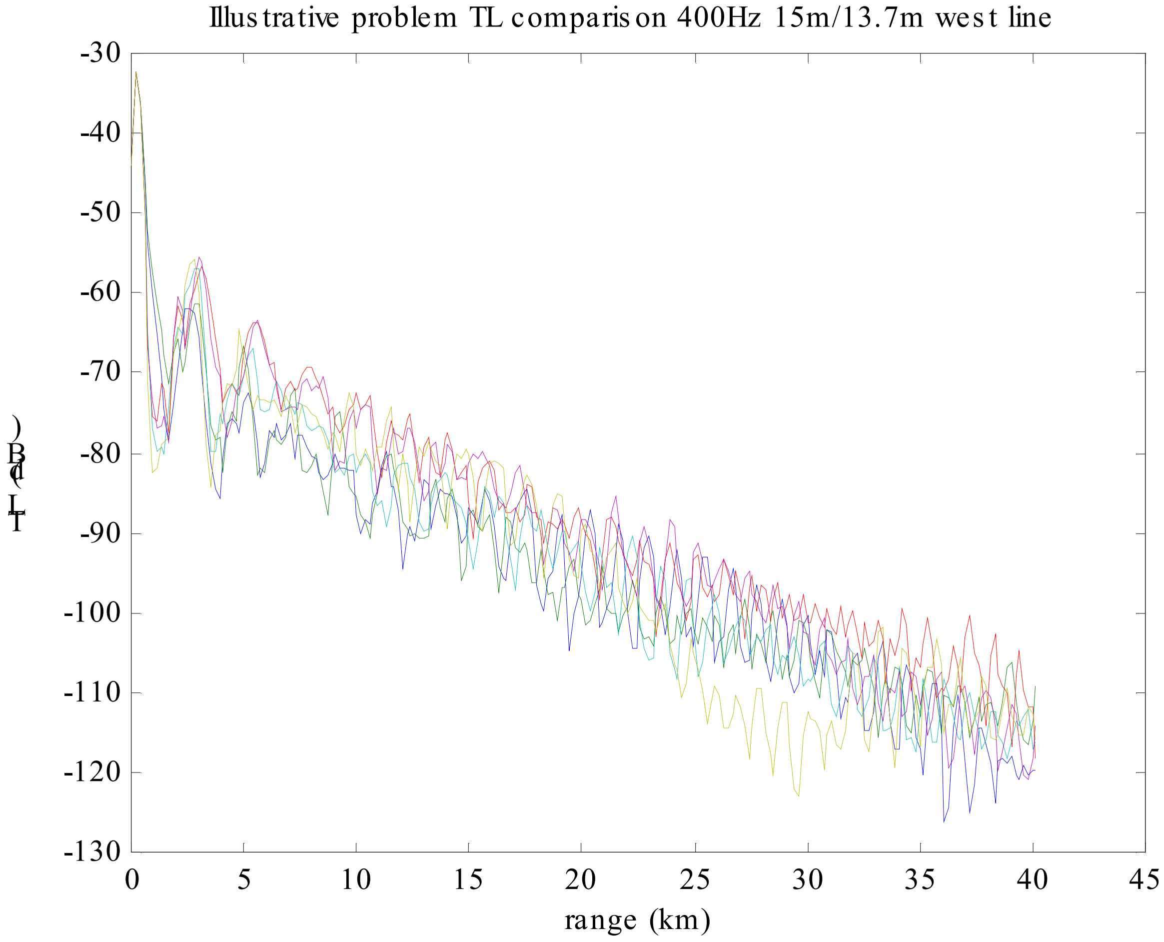


Profiles shifted by 10m/sec for display
Range separation is 2.4km

SVP mean gradient over depth from 0-25m for each of 6 days



Variability - Smoothed TL for each of 6 days on westline at receiver depth 13.7m



Conclusion of this comparison

Assumption of fully independent random sound speed variations representing measurement uncertainty is not adequate to describe the effect of gradient changes

> Interaction with the bottom introduces major differences in level, and the gradient controls this interaction.
>
> Changes in major interference patterns like Lloyd's mirror are not predicted

Definition of uncertainty and variability will be mission, frequency and system dependent and methods must be developed to model both

Modeling improvements

•Inclusion of depth correlation function for speeds

•New approach may be derivative of pressure field with respect to the extremum layer depth- bathymetry and bottom attenuation must play a role

•Inputs required

gradient statistics, horizontal and vertical

layer position and vertical displacement

•Outputs required

range of coherence effects (location of caustics)

level variations

UNCERTAINTY IN PROPAGATION LOSS CALCULATIONS

Diana McCammon
Applied Research Laboratory
Pennsylvania State University

This presentation supports the illustrative problem selected by the ONR DRI to demonstrate the measurement or simulation, propagation, and usage of uncertainty in the underwater environment. The environmental inputs are obtained from two sources: the measured Primer experiment and the Harvard model simulator. The resulting propagation loss calculations show the contrast between an entirely random error in sound speed and a daily variability of the profile.

Figure 1 displays the sources of uncertainty that are introduced in setting up a propagation model. The Primer bathymetry is illustrated in Figure 2, and the various measured speed profiles are shown in Figure 3 for each of the six days of the experiment along the west line of the Primer area. Internal smoothing and bottom characteristics are discussed in Figures 4 and 5, and a sample transmission loss curve showing the quarter nautical mile smoothing is shown in Figure 6. Figure 7 displays the smoothed transmission loss for each of the six days, for a single geometry, while Figure 8 displays the mean level in a full field of the transmission loss. (dB mean over the six days) and Figure 9 displays the standard deviation in this mean level full field. A single geometry is chosen to illustrate the same two quantities (mean and standard deviation) in Figure 10. This represents the uncertainty in prediction of the propagation loss due to the six day variability in the environment.

To examine the propagation of a random error in an individual sound speed value, we can use standard normal probability techniques. Figure 11 displays the confidence bounds for an assumed random intensity. A table of typical confidence levels is shown in Figure 12. The level of 68% (alpha=.32) is chosen to produce confidence bounds that would correspond to one standard deviation.

Figures 13-16 sketch the mathematics. The sound speed is decomposed into a mean value and a zero-mean random component, $c(1+x)$. The intensity of this perturbed environment is expanded in a Taylor series about x, and using the mean and standard deviation of the sound speed, the confidence bounds are computed. The derivative of the pressure with respect to speed perturbation is found from the split step Fourier parabolic equation by the chain rule. Errors are computed individually for each depth and the RMS sum is formed. Figure 17 displays the mean and standard deviation of the speed used as an input to this random error propagation model. Figures 18 and 19 contrast the mean and standard deviation of the transmission loss as computed by the statistical analysis of the measured variability and by the random error propagation model. The mean fields are similar but the error field is very much greater for the measured six day variability. Figures 20 and 21 are just different representations of this result.

Figure 22 repeats the measured profiles to illustrate the random looking nature of the changes, however, in Figure 23, the average gradient over 25m from each of the six days is plotted and the 6th day is seen to have twice the negative gradient as some of the other days. Figure 24 repeats the Primer westline transmission loss vs range for a fixed geometry, where the bottom yellow curve that dips very low from 25-30km is found to be that from the 6th day, confirming that the steeper negative gradient caused more interaction with the bottom and more subsequent loss.

Figures 25 and 26 present the conclusion of this propagation comparison, which is that the assumption of a fully independent random sound speed variation is not adequate to describe the effect of gradient changes. The interaction of the sound field with the bottom introduces major differences in level and the gradient controls this interaction. Individually randomly varying speeds at each depth do not reproduce this effect. In the same vane, changes in the major interference patterns of the Lloyd's mirror are not predicted. The differences between uncertainty and variability will be mission and situation dependent, and so methods need to be developed to predict the propagation of error in both cases.

Modeling improvements that might lead to a better prediction of the variability error include using the depth correlation of the speeds, and examining the effect of an randomly varying extremum in the profile, that is, randomly varying layer depths. It is recognized that any mathematical approach must also include the bathymetry and the bottom attenuation. This may require some new statistical quantities for description of the sound field, including horizontal and vertical gradient statistics, and layer position and displacement statistics. Finally, it is noted that to support many of the fleet applications that require propagation loss, the output of an uncertainty propagation calculation will be required to specify not only the level variations to be expected, but the range shifts of the major coherence effects such as the Lloyd's mirror, the shadow zones and the convergence zones as well.